Gamma-ray Spectrometry

Training Workshop on
Applications of Gamma-ray Spectrometry
to Environmental Samples

Angular Correlations

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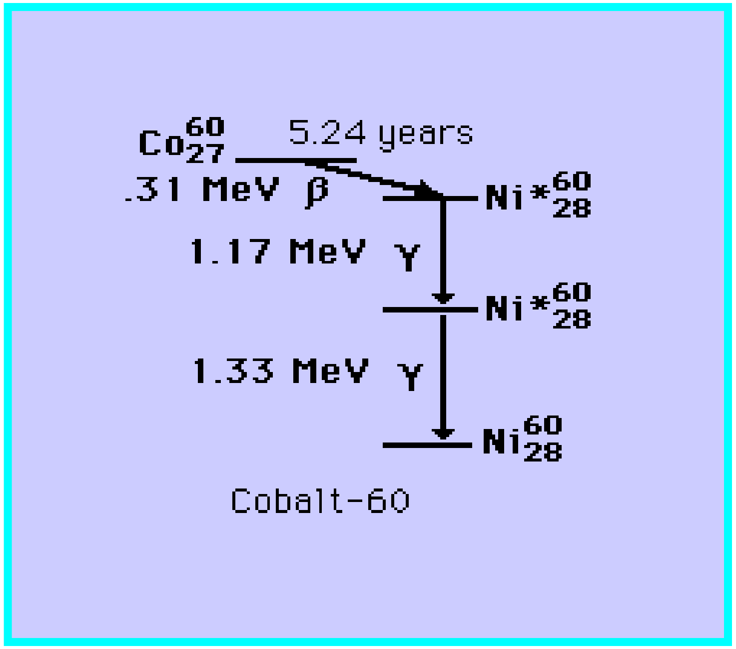
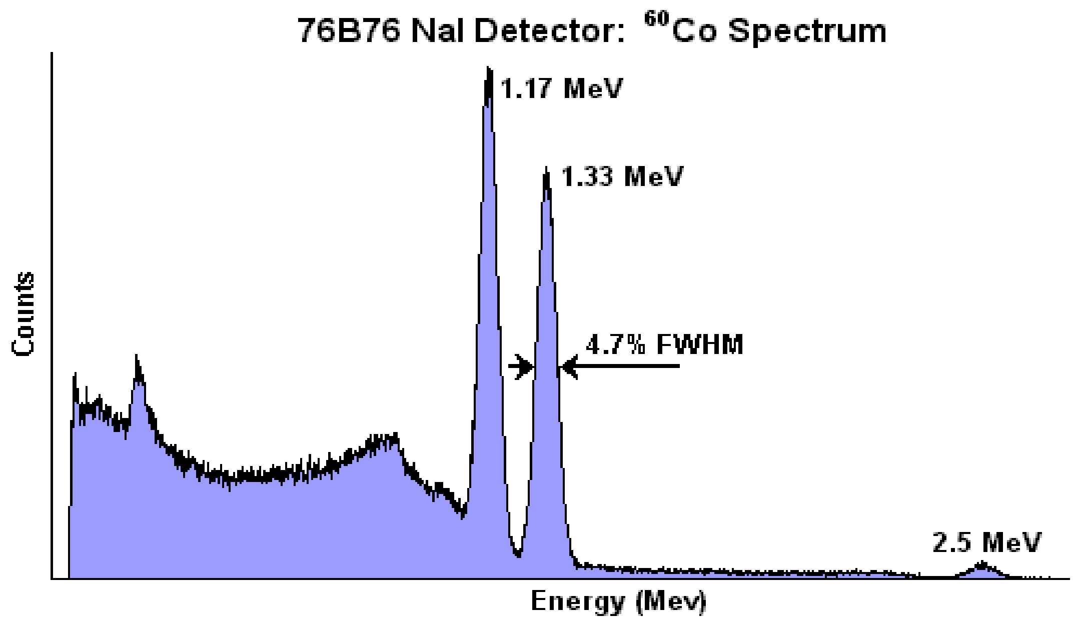
Coincidence-summing effect with Co-60





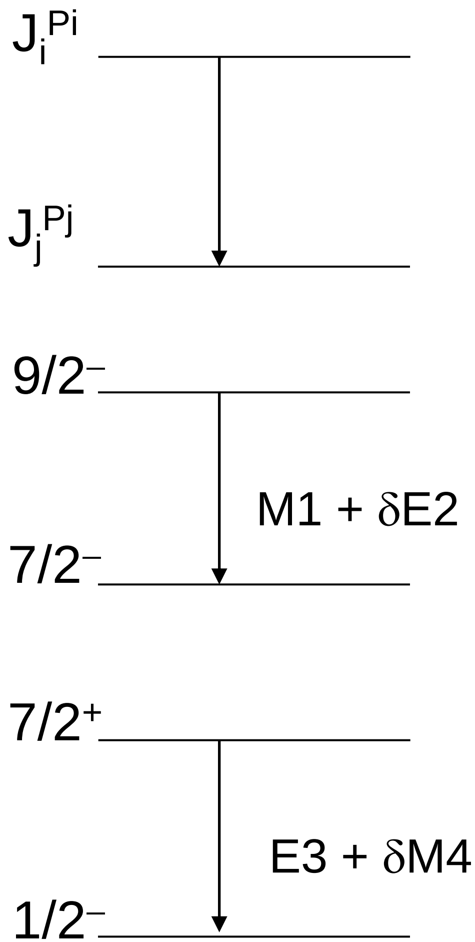
Gamma transition

- Gamma rays are extremely high frequency electromagnetic radiation
- The radiation is quantized into photons
- Excited nuclear states decay into lower-lying ones by gamma emissions
- The EM operator acts between the initial and final state of the nucleus
- The operator can be developed into a multipole series
- This is analogous to moments of classical charge and current distribution
- Emission probability of a photon depends on the angle θ between the nuclear spin and the photon direction

Gamma transition

- An oscillating electric dipole d consists of charges q^+ and q^- set at a varying distance z : $d = q\, z_0 \cos(\omega t)$
- Magnetic dipole is a current loop
- There is are no monopole transitions
- The multipole order L defines the transition
- Transition multipolarity is 2^L : L=1 dipole, L=2 quadrupole, …
- Angular distribution of 2^L-pole radiation is Legendre polynomial $P_{2L}(\cos\theta)$
- The parity of the radiation field is $(-1)^{L+1}$ for magnetic and $(-1)^L$ for electric transitions
- Lower multipole transitions are dominant
- For a given L, electric transition is dominant

Selection Rules



$$|J_i - J_f| \leq L \leq |J_i + J_f|$$

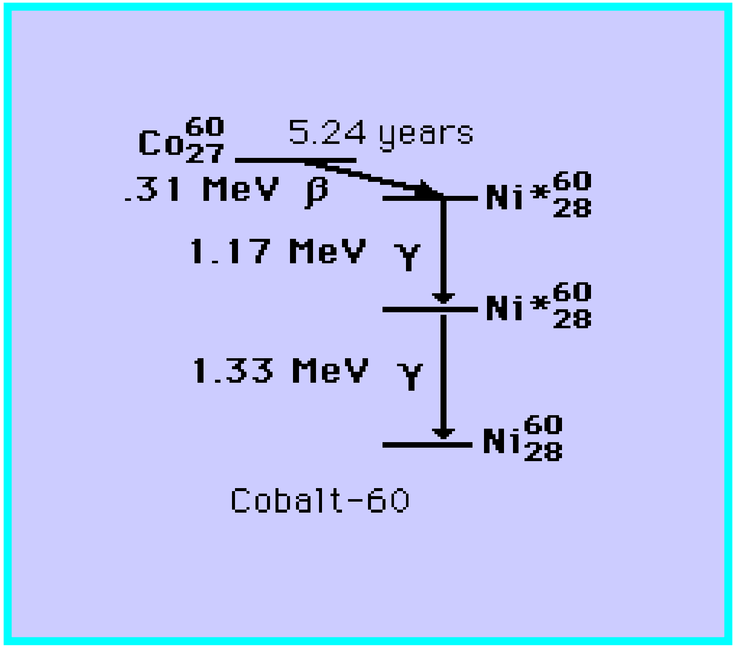
$$|J_i - J_f| = 1 \; ; \; P_i P_f = +$$

$$|J_i - J_f| = 3 \; ; \; P_i P_f = -$$

Gamma transition

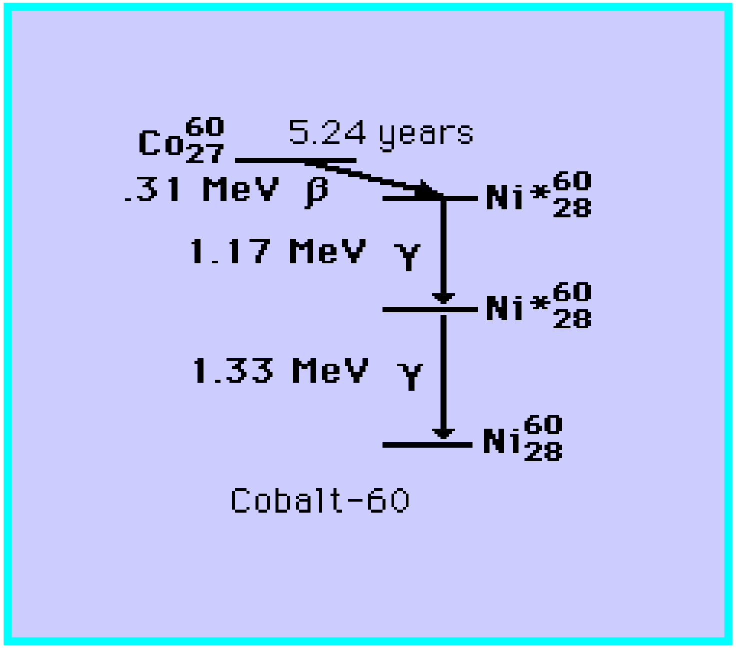
- $|J_i - J_f| \leq L \leq |J_i + J_f|$
- No change in parity tween the initial and final state of the nucleus: even electric, odd magnetic transition
- Change in parity : odd electric, even magnetic transitions
- $J_i = J_f$, $L = 1,2,\ldots |J_i + J_f|$, no $L = 0$ (monopole)
- $J_i = 0$ or $J_f = 0$: one single multipolarity
- $J_i = J_f = 0$: forbidden, internal conversion instead

Two step cascade



- Nuclear moments not oriented at room temperature and $B = 0$
- First gamma ray: preferential direction (z-axis) not defined
- First gamma ray emitted isotropically
- First gamma ray **defines** the preferential direction (z-axis)
- Second gamma ray emitted according to the multipolarity of the transition! → $P_{2L}(\cos\theta)$

The case of Co-60

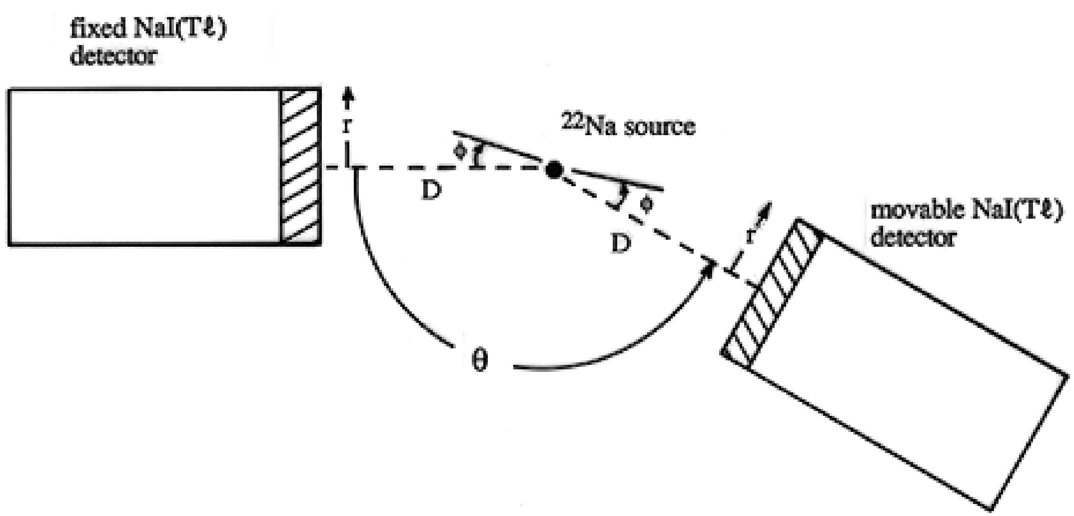


- Transitions are $4^+ \rightarrow 2^+ \rightarrow 0^+$
- Selection rules allow only pure E2 transition for gamma 2
- Gamma 1 has negligible admixture of M3 and higher multipoles
- Angular distribution $W(\theta)$ of the direction of the second gamma with respect to the first:

$$W(\theta) = 1 + \sum_{k=1}^{L} a_{2k}\,(\cos\theta)^{2k}$$

- Co-60: $a_2 = 1/8$, $a_2 = 1/24$

Angular Correlations Measurement



- Measuring *W* (θ) brings information on the multipolarity of transitions
- We can determine spins and parities of nuclear states
- To distinguish between electrical and magnetic transitions, we need polarization measurements

Angular correlations and true coincidence summing

- Consider Co-60 decay
- Second gamma ray not emitted isotropically – more likely to hit the detector than if there were no angular correlations!
- Therefore, enhanced summing-in and summing-out by a factor *w*

$n_1 = A\varepsilon_1(1 - w\varepsilon_{t2})$
$n_2 = A\varepsilon_2(1 - w\varepsilon_{t1})$
$n_{12} = Aw\varepsilon_1\varepsilon_2$

Approximation: First gamma ray emitted straight down the detector axis

$$w = \int_0^{\alpha} W(\theta)\,\sin\theta\,d\theta \Big/ \int_0^{\alpha} \sin\theta\,d\theta$$

- Exact approach : Monte Carlo simulations with the complete decay scheme and sampling of angular correlations

Angular correlations and true coincidence summing

Angular correlations

$n_1 = A\varepsilon_1\left(1 - w\varepsilon_{t2}\right)$
$n_2 = A\varepsilon_2\left(1 - w\varepsilon_{t1}\right)$
$n_{12} = Aw\varepsilon_1\varepsilon_2$
$A = Cn_1 / \varepsilon_1$
$C = 1/\left(1 - w\varepsilon_{t2}\right)$

No angular correlations

$n_1 = A\varepsilon_1\left(1 - \varepsilon_{t2}\right)$
$n_2 = A\varepsilon_2\left(1 - \varepsilon_{t1}\right)$
$n_{12} = A\varepsilon_1\varepsilon_2$
$A = Cn_1 / \varepsilon_1$
$C = 1/\left(1 - \varepsilon_{t2}\right)$

Angular correlations and true coincidence summing

Angular correlations

$\varepsilon_{t2} = 0.1$
$w = 1.05$
$n_{12} = 1.05 A \varepsilon_1 \varepsilon_2$
$A = C n_1 / \varepsilon_1$
$C = 1.12$

$A = \pi r$

No angular correlations

$\varepsilon_{t2} = 0.1$
$w = 1.00$
$n_{12} = A \varepsilon_1 \varepsilon_2$
$A = C n_1 / \varepsilon_1$
$C = 1.11$

Angular correlations and true coincidence summing

- Only few radionuclides are affected – triple and double cascades, Ru-106
- Large sample-detector distance: w can reach 1.1
  (Co-60, 10 cm, point source), but coincidence effects are small
- Close geometry: $W(\theta)$ is averaged out, w close to unity
- Intermediate sample-detector distances: the effect does not exceed a couple of per cent with regard to the coincidence summing-out correction factor
- Summing in: effect can be significant, but summing-in rarely dominates
- Therefore: no need to consider angular correlations in environmental measurements
- A study: M. Roteta, E. Garcia-Toraño, NIMA 369 (1996) 665-670